Variance-Based Analysis of Global Criteria Importance in the ESP-COMET Method

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Abstract

Sensitivity analysis is a critical component of Multi-Criterion Decision Analysis (MCDA), enabling the evaluation of how variations in input data influence decision outcomes. Although traditional techniques, such as scenario analysis and stability intervals, can enhance robustness, they often fail to capture the full variability inherent in the decision model. To overcome this limitation, we propose a novel framework that applies Sobol sensitivity indices to assess the global importance of criteria in MCDA, with a particular focus on models constructed using the Characteristic Objects Method (COMET). Unlike standard approaches that perturb the decision matrix, our method evaluates the sensitivity of the decision model as a whole. We demonstrate its effectiveness through controlled experiments, including hybrid configurations such as Expected Solution Point (ESP) with COMET, and use first-order ($S1$) and total effect (ST) Sobol indices. The results show that the variance-based sensitivity analysis provides valuable, complementary insights into the global behavior of the model and the influence of individual criteria. This study introduces a robust analytical tool that enhances the interpretability and methodological depth of decision support systems in MCDA.

Keywords: MCDA, Sensitivity Analysis, Sobol' indices, COMET.

1. Introduction

Sensitivity analysis (SA) is a key component of Multi-Criteria Decision Analysis (MCDA), enhancing transparency, identifying influential factors, and supporting adaptive decision-making by assessing how sensitive outcomes are to changes in inputs such as criteria weights, scores, or assumptions [3], [16]. Various complementary approaches are used in SA, including scenario analysis, which explores how decision outcomes vary under different assumptions [1], and the determination of stability intervals, which define input ranges (for example, criteria weights) within which the classification of alternatives remains unchanged, ensuring the reliability of results [17]. Software tools such as D-Sight further support this process by enabling real-time, interactive sensitivity analysis, allowing users to dynamically adjust parameters and immediately observe the effects on decision outcomes [6].

Nevertheless, MCDA sensitivity analysis rarely involves a comprehensive assessment of the total variance of the model, as is done in global sensitivity analysis methods, which quantify the contribution of individual inputs to the overall uncertainty of the model output. Together, the techniques applied in MCDA provide a practical framework for understanding how robust a solution is to uncertainties and variations in model input, although full variance-based decompositions are typically not performed [17].

However, some efforts have been made to incorporate variance-based techniques into MCDA frameworks. In particular, adaptations of global sensitivity analysis methods, such as the Sobol approach, have been proposed to better assess the robustness and sensitivity of ranking results to changes in model input and stakeholder preferences. Although these techniques were originally designed for simulation models, recent developments demonstrate their potential to enhance the understanding of uncertainty and variability in MCDA applications [10].

In this work, we propose the use of Sobol' indices for Global Criteria Significance Sensitivity Analysis, where the focus is not on perturbing the decision matrix, but rather on analyzing the sensitivity of the entire decision model. We present a series of synthetic experiments to demonstrate the differences between this approach and the traditional global sensitivity analysis used within the COMET method. The experiments will also include hybrid approaches, such as ESP-COMET, to illustrate broader applicability. In the analysis, we employ both first-order (S1) and total-effect (ST) Sobol indices. This approach offers a valuable complement to basic criteria' importance analysis and provides additional insights into model behavior. The main contribution of this study is a framework that enables the use of Sobol's indices to evaluate decision models identified through the COMET method.

The remainder of the paper is organized as follows. Section 2 reviews related works; Section 3 presents the proposed methodology; Section 4 describes the case study and discusses the results; Section 5 concludes the paper and outlines future research directions.

2. Related works

Sensitivity analysis is vital to understand complex models under uncertainty, enabling robust decisions by quantifying how input variability affects output. Among global methods, the Sobol approach stands out due to its variance-based foundation and applicability across disciplines [2]. In MCDA, where weights and performance values are often uncertain, global techniques like Sobol or Fourier amplitude sensitivity testing (FAST) are preferable to local ones, as they explore the full input space. These methods decompose the variance of the output to identify the influential inputs, providing insight into the sensitivity and structure of the model [11]. To address the lack of formal SA in MCDA rankings, Ganji et al. adapted the Sobol method using rank equivalence boundaries (REB). Their framework consists of REB equations, uniform sampling, and sensitivity indices. It identified stable versus unstable rankings in water management scenarios and highlighted key parameters [7].

The classical Sobol method assumes independent, uniformly distributed inputs, which often does not hold in practice. Recent modifications accommodate non-uniform distributions by transforming variables and correcting joint densities, enhancing its real-world applicability [4]. In environmental decision-making, for example, in reservoir operations, the Sobol analysis filters noninfluential variables, guiding efficient multiphase optimizations under policy constraints [14]. Similarly, in agriculture, it supports sustainable fertilization strategies by modeling plant–soil interactions using free ion activity model (FIAM), Fuzzy-TOPSIS (Technique for the Order of Prioritisation by Similarity to Ideal Solution), and Monte Carlo methods, identifying variables critical for potassium release and yield [8]. Overall, Sobol-based SA is a powerful yet underused tool, especially in rule-based models. Its broader adoption could improve transparency, parameter relevance, and model interpretability in diverse applications.

3. Methodology

This section outlines the proposed approach. It begins with the ESP-COMET, which enhances the classical COMET method [12] by incorporating expert-defined reference points. The next part introduces the use of global weights within COMET to account for the varying importance of the criteria. Finally, the methodology includes a sensitivity analysis based on the Sobol's method and Saltelli sampling scheme to evaluate the influence of each criterion on the results.

3.1. ESP-COMET

The ESP-COMET extends the classical COMET method by integrating expert knowledge in the form of Expected Solution Points (ESP) [5]. The expert provides one or more ESPs, each defined as a vector whose dimensionality corresponds to the number of criteria in the decision problem. These ESPs represent expected, ideal, or desirable solutions in the criteria space [13].

The ESP-COMET approach was selected as an extension of COMET in this methodology because it enables intuitive and purposeful shaping of the decision surface geometry through precise specification of reference points aligned with expert expectations. Each characteristic object (CO) is evaluated based on its distance from the provided ESPs. The distance function can be customized to reflect the nature of the criteria and the decision-making context. When multiple ESPs are defined, an aggregation function is used to combine the distances from different ESPs into a single measure for each CO. This approach is fully implemented in the `pymcdm` library, which provides ready-to-use functionality for defining ESPs, selecting distance metrics, and aggregating results [9].

3.2. Global weights in the COMET method

To determine global weights in the COMET method, the preference values assigned to all characteristic objects are analyzed [15]. A linear regression model is then fitted to approximate the relationship between the criteria and the computed preferences. The resulting model takes the following form:

$$\hat{P} = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_m x_m, \tag{1}$$

where $\hat{P}$ represents the estimated preference value, and α_i are the regression coefficients corresponding to the criteria x_i. The global weight w_i for each criterion is calculated as the normalized absolute value of its respective coefficient.

$$w_i = \frac{|\alpha_i|}{\sum_{i=1}^{m} |\alpha_i|}. \tag{2}$$

Including global weights into the COMET framework improves the evaluation process by introducing an explicit representation of the importance of the criteria, while preserving the method's data-driven and locally sensitive structure.

3.3. Sobol' sensitivity analysis using the Saltelli scheme and the COMET method

The complete proposed algorithm has been presented in Algorithm 1. In the first step, we apply the Saltelli scheme for global sensitivity analysis to estimate Sobol indices for the `COMET` model. This variance-based approach quantifies the contribution of each input variable to the total output variance, considering both first-order effects (direct influence) and total-order effects (including interactions). The procedure begins with the generation of two independent quasirandom sampling matrices A and B of size $N \times d$, where N is the number of samples and d is the number of input variables. Each row in these matrices corresponds to one model evaluation. The model is evaluated separately on both matrices, producing output vectors y_A and y_B. For each input variable i, a new matrix $A_B^{(i)}$ is created by replacing the i-th column of matrix A with the corresponding column from B. The model is then evaluated on each $A_B^{(i)}$, yielding outputs $y_{A_B^{(i)}}$. The total output variance V is computed from the evaluations on A. The first-order Sobol' index S_i is calculated as the covariance between y_B and the difference $y_{A_B^{(i)}} - y_A$, normalized by the total variance. The total-order index ST_i is estimated as the expected squared difference between y_A and $y_{A_B^{(i)}}$, also normalized by V. To assess the reliability of the sensitivity estimates, we compute the associated confidence measures S_{conf} and ST_{conf}, based on the empirical standard deviations of the respective estimators across the sample.

Algorithm 1: Numerical Estimation of Sobol' Indices for a `COMET` Model (Saltelli Scheme)

Input: `COMET`$(\cdot)$ – model evaluator,
N – sample size,
d – number of input variables
Output: First-order indices S_i and their uncertainties $S1_{\text{conf}}$,
Total-order indices ST_i and their uncertainties ST_{conf}, $i = 1, \ldots, d$
Step 1. Generate two independent quasi-random matrices
$A, B \in [0,1]^{N\times d}$ (e.g. Sobol sequences).;
Step 2. Evaluate the model on the base matrices;
$y_A \leftarrow \texttt{COMET}(A), \quad y_B \leftarrow \texttt{COMET}(B)$;
Step 3. for $i \leftarrow 1$ **to** d **do**
Build $A_B^{(i)}$ by replacing column i in A with column i from B;
Compute $y_{A_B^{(i)}} \leftarrow \texttt{COMET}(A_B^{(i)})$;
Step 4. Estimate the total output variance $V = \frac{1}{N}\sum_{j=1}^{N}\big(y_A^{(j)}\big)^2 - \Big(\frac{1}{N}\sum_{j=1}^{N} y_A^{(j)}\Big)^2$.
Step 5. for $i \leftarrow 1$ **to** d **do**

$$\text{First-order index } S1(C_i) = \frac{\frac{1}{N}\sum_{j=1}^{N} y_B^{(j)}\big(y_{A_B^{(i)}}^{(j)} - y_A^{(j)}\big)}{V};$$

$$\text{Total-order index } ST(C_i) = \frac{\frac{1}{2N}\sum_{j=1}^{N}\big(y_A^{(j)} - y_{A_B^{(i)}}^{(j)}\big)^2}{V}.$$

$$S1_{\text{conf}}(C_i) = \sqrt{\frac{1}{N(N-1)}\sum_{j=1}^{N}\left(\frac{y_B^{(j)}(y_{A_B^{(i)}}^{(j)} - y_A^{(j)})}{V} - S1_i\right)^2};$$

$$ST_{\text{conf}}(C_i) = \sqrt{\frac{1}{N(N-1)}\sum_{j=1}^{N}\left(\frac{(y_A^{(j)} - y_{A_B^{(i)}}^{(j)})^2}{2V} - ST_i\right)^2}.$$

4. Study cases

This section presents selected two-dimensional models identified using the ESP-COMET approach as a proof of concept. For each model, results based on global importance weights are compared with the corresponding first-order ($S1$) and total-effect (ST) Sobol sensitivity indices. The examples highlight the role of these indices in interpreting model behavior from a global perspective and demonstrate the potential of integrating Sobol analysis into the COMET family methods to enhance its analytical and interpretative potency.

The first case illustrates a situation in which one of the criteria, criterion C_1, shows a significant influence but has no impact on the final result from a global perspective, as calculated in Section 3.2. The two-dimensional model used in this example is shown on the left side of

Figure 1. This discrepancy highlights the need to incorporate global sensitivity measures into the interpretation of the model. Sobol indices, in particular, should be used to complement the global analysis by quantifying both the individual and interaction effects of input criteria throughout the input space. Focusing only on direct effects can lead to a misleading understanding of the overall importance of the criterion.

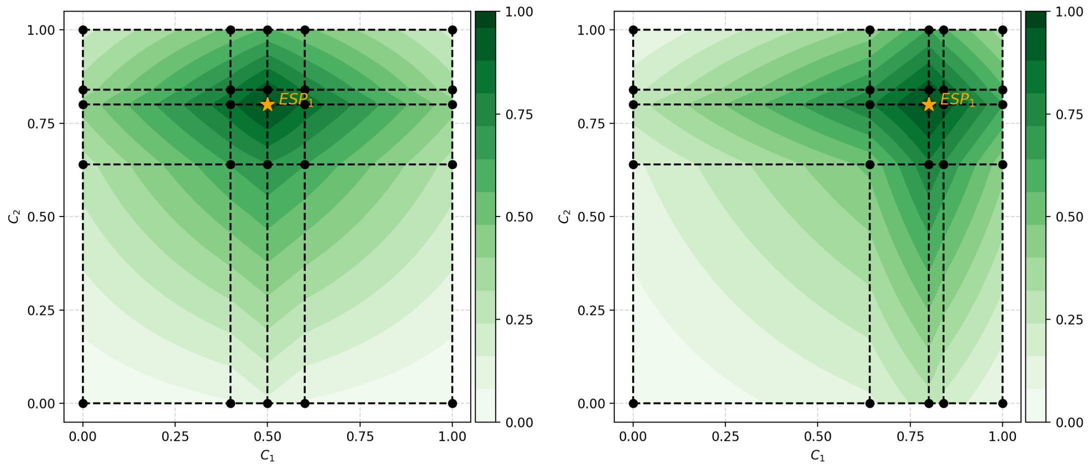

Fig. 1. Two-dimensional models for criteria C_1 and C_2 with the evaluation grid and ESP point: $[0.5, 0.8]$ (left) and $[0.8, 0.8]$ (right).

The results in Table 1 illustrate the relationship between the global importance weights of the ESP-COMET model and the sensitivity measures derived from the Sobol method. Despite the first criterion (C_1) exhibiting non-zero Sobol indices, first order $S1 = 0.2291$ and total effect $ST = 0.2614$, its weight of global importance is $w_1 = 0.0000$. This indicates that although the estimated global weight of C_1 suggests that it does not affect the final decision outcome in the ESP-COMET model (i.e. it is zero at the evaluated point), the criterion still contributes to the variability of the model output across the input space. This influence, which is not captured by importance weights alone, is revealed through variance-based sensitivity analysis using Sobol indices.

Table 1. Comparison of ESP-COMET global importance weights (w) with Sobol sensitivity measures for the ESP point $[0.5, 0.8]$.

	w	$S1$	ST	$ST - S1$	$S1_{\text{conf}}$	ST_{conf}
C_1	0.0000	0.2291	0.2614	0.0323	0.0452	0.0248
C_2	1.0000	0.7380	0.7680	0.0300	0.0610	0.0540
$\sum$	1.0000	0.9671	1.0294	0.0623	0.1062	0.0789

In contrast, the second criterion (C_2) exhibits dominant global importance ($w = 1.0000$), supported by high Sobol indices: $S1 = 0.7380$ and $ST = 0.7680$. Although these values are significantly higher than those for C_1, they are not close to 1, reflecting a more realistic estimate of the actual influence of the criterion. The small differences between ST and $S1$ for both criteria ($ST(C_1) - S1(C_1) = 0.0323$ and $ST(C_2) - S1(C_2) = 0.0300$) suggest limited interaction effects, indicating that most of the sensitivity is attributable to main effects rather than higher-order interactions.

The sum of the $S1$ values (0.9671) and the ST values (1.0294) being close to 1 further supports the interpretation that the model is largely additive, with limited mutual dependency between criteria. Furthermore, the relatively low confidence intervals for the Sobol indices ($S1_{\text{conf}}(C_1) = 0.0452$, $S1_{\text{conf}}(C_2) = 0.0610$, $ST_{\text{conf}}(C_1) = 0.0248$, $ST_{\text{conf}}(C_2) = 0.0540$)

indicate that the sensitivity estimates are stable and reliable. This case shows the importance of combining global importance weights with variance-based sensitivity analysis. Although the ESP-COMET model assigns zero weight to the criterion C_1, the Sobol indices indicate its significant influence on output variability. In the two-dimensional case, as shown in Figure 1 (left), the discrepancy is easy to spot, but in higher-dimensional models it would be much harder to detect. Therefore, the proposed approach helps reveal hidden influences that might otherwise be overlooked.

The second case study focuses on the ESP point $[0.8, 0.8]$. On the right side of Figure 1, the two-dimensional decision surface reflects equal contributions from criteria C_1 and C_2. This example demonstrates that the process of deriving global weights can capture the same variance-driven influences identified by global sensitivity measures. The first order indices ($S1 = 0.4830$ for C_1, 0.4822 for C_2) and the total effect indices ($ST = 0.5190, 0.5175$) closely match the COMET weights of $w = 0.5$ (Table 2). The interaction terms remain small ($ST - S1 < 0.04$), indicating that the main effects dominate. In addition, $\sum S1 = 0.9652$ and $\sum ST = 1.0365$ confirm the near-additive behavior of the model, while the confidence intervals ($S1_{\text{conf}} \leq 0.0672$, $ST_{\text{conf}} \leq 0.0481$) underscore the robustness of these estimates. Together, these metrics demonstrate that, at this ESP point, COMET's rule-based importance weighting provides a faithful approximation of the true global sensitivity profile.

Table 2. Comparison of ESP-COMET global importance weights (w) with Sobol sensitivity measures for the ESP $[0.8, 0.8]$.

	w	$S1$	ST	$ST - S1$	$S1_{\text{conf}}$	ST_{conf}
C_1	0.5000	0.4830	0.5190	0.0360	0.0672	0.0481
C_2	0.5000	0.4822	0.5175	0.0354	0.0608	0.0437
$\sum$	1.0000	0.9652	1.0365	0.0714	0.1280	0.0918

In the third case, we explore the global behavior of the model by evaluating two example ESP vectors: $[0.2, 0.4]$ and $[0.6, 0.7]$. By incorporating more than one ESP point, we better approximate real-world decision-making scenarios, where multiple expert evaluations are typically involved. This example illustrates how the two approaches capture different aspects of the significance of the criteria across the input space. Figure 2 presents the evaluated ESP points on the two-dimensional decision surface.

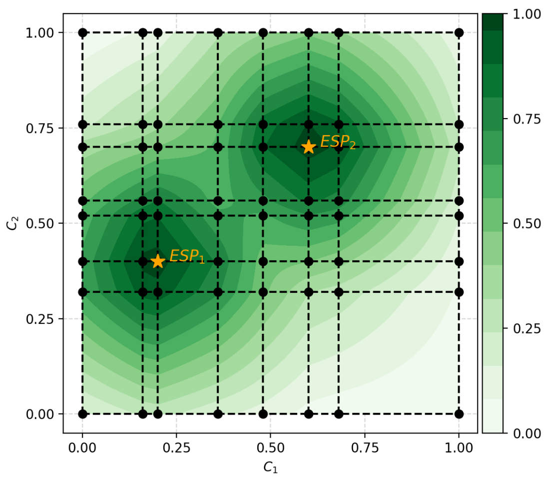

Fig. 2. Two-dimensional model for criteria C_1 and C_2 with the evaluation grid and two ESP points: $[0.2, 0.4]$ and $[0.6, 0.7]$.

Table 3 presents the global importance weights and Sobol sensitivity indices computed for this multipoint configuration. The ESP-COMET model assigns a greater global significance to criterion C_1 ($w_1 = 0.6533$), indicating its dominant role in determining the final decision outcome in all evaluated points. In contrast, the Sobol indices show that the criterion C_2 plays a stronger role in explaining the variability of the output across the whole input space. Specifically, criterion C_2 has higher values for both the first-order index ($S1(C_2) = 0.3538$) and total-effect index ($ST(C_2) = 0.7747$), compared to criterion C_1 ($S1(C_1) = 0.2298$, $ST(C_1) = 0.6292$).

Table 3. Comparison of ESP-COMET global importance weights (w) with Sobol sensitivity measures for two ESP points $[0.2, 0.4]$ and $[0.6, 0.7]$.

	w	$S1$	ST	$ST - S1$	$S1_{\text{conf}}$	ST_{conf}
x	0.6533	0.2298	0.6292	0.3994	0.0708	0.0568
y	0.3467	0.3538	0.7747	0.4209	0.0796	0.0701
$\sum$	1.0000	0.5836	1.4039	0.8203	0.1505	0.1269

The notable differences between $S1$ and ST values for both criteria (0.3994 for C_1, 0.4209 for C_2) indicate strong interaction effects, meaning that the contribution of each criterion cannot be fully understood without considering how they interact with one another. In this context, while the COMET weights highlight the dominance of C_1 in producing the final evaluations, Sobol indices reveal that C_2 has a greater impact on the variability of the output across the global input space. This case appears to be the most typical for decision models, revealing different aspects of criteria importance depending on the chosen analysis method. It confirms the value of combining ESP-COMET weights with Sobol sensitivity analysis for a more comprehensive interpretation.

5. Conclusions

This study introduces a novel framework for the analysis of global sensitivity within the COMET method by applying variance-based Sobol indices to evaluate the importance of criteria in the MCDA. Unlike other local approaches that focus on perturbing the decision matrix or specific input values, the proposed method analyzes the sensitivity of the entire decision model. This enables a more holistic and globally informed interpretation of how individual criteria influence results across the entire input space.

We examined extreme, consistent, and typical two-dimensional cases to assess how global weights and Sobol indices complement each other. While global weights reflect direct contributions to outcomes, Sobol indices capture output variability, including non-linearities and interactions. The approach is particularly useful in high-dimensional problems, where local interpretations are difficult. Aggregating results across profiles helps smooth irregularities and better identify key criteria, enhancing interpretability and reliability. Future research should focus on extending this approach to decision problems with higher dimensionality and developing advanced visualization tools for multidimensional sensitivity analysis.

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